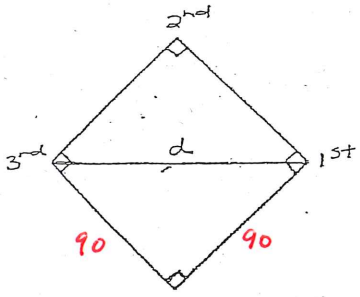
Special Right Triangles

Below is a diagram of a baseball diamond. The distance between each base is 90 feet. How far is it between 1st base and 3rd base? Let the Pythagorean Theorem be of your assistance.



d = 127.279 ft

*If you draw a line from 1st to 3rd base, what are the measures of the three angles in the two triangles that are created by the line?

45, 45, 90

* Are the two triangles created congruent? Why or why not?

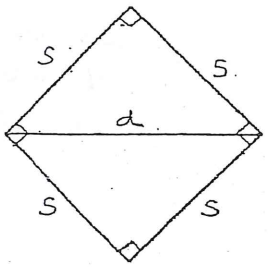
Yes - SAS

$90^2 + 90^2 = c^2$

$8100 + 8100 = c^2$

$16,200 = c^2$

$127.279 \text{ ft} = c$



In general, what can be said about the lengths of the sides of a 45°-45°-90° triangle? Once again, use the Pythagorean Theorem to help you out.

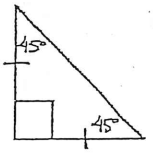
$s^2 + s^2 = d^2$

$2s^2 = d^2$

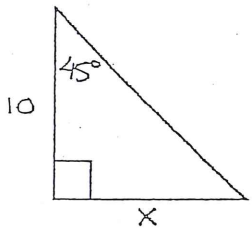
$\sqrt{2s^2} = d$

$\sqrt{2}s = d$

Theorem: In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.

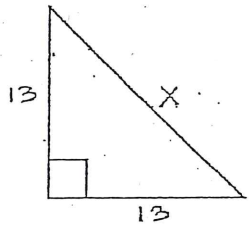


Ex. #1: Find x.



x = 10

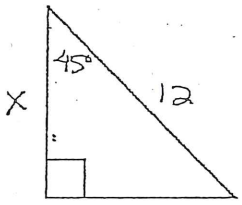
Ex. #2: Find x.



x = $13\sqrt{2}$

18.385

Ex. #3: Find x.



$c = a\sqrt{2}$

$\frac{12}{\sqrt{2}} = \frac{a\sqrt{2}}{\sqrt{2}}$

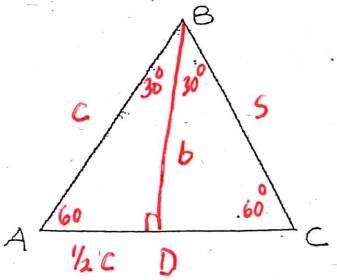
x = $\frac{12}{\sqrt{2}}$

8.485

Below is an equilateral triangle. From this information we know that the sides are congruent and the angles are all equal. Draw in altitude $\overline{BD}$. What are the angles of the two triangles that you have just created?

30°, 60°, 90°. Label the altitude as "a". If $\overline{AB}$ is "s", label segments $\overline{BC}$, $\overline{AD}$, and $\overline{DC}$.

Use the Pythagorean Theorem to find the "a" in terms of the lengths of the other sides of the triangle.



a = $\frac{\sqrt{3}}{2}c$

$b^2 + \left(\frac{1}{2}c\right)^2 = c^2$

$b^2 + \frac{1}{4}c^2 = c^2$

$b^2 = \frac{3}{4}c^2$

$b = \sqrt{\frac{3}{4}c^2}$

$b = \sqrt{\frac{3}{4}}\,c$

$b = c\sqrt{\frac{3}{4}}$

$b = c\frac{\sqrt{3}}{\sqrt{4}}$

$b = c\frac{\sqrt{3}}{2}$

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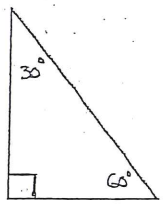
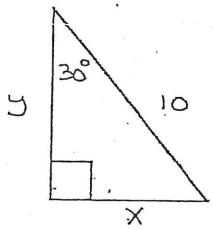
So, the sides of a 30°-60°-90° triangle are $\frac{1}{2}c$ (Short Leg), $\frac{\sqrt{3}}{2}c$ (Long Leg), c (Hypotenuse).

Now, get rid of the fractions by multiplying all three measurements by the common denominator. The following theorem uses these measurements.



Theorem: In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

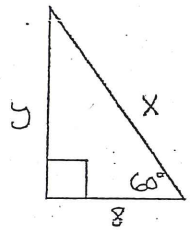
Ex. #4: Find x and y.



x = 5

y = $5\sqrt{3}$

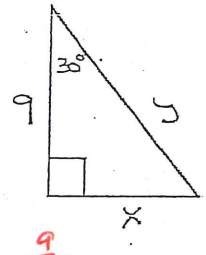
Ex. #5: Find x and y.



x = 16

y = $8\sqrt{3}$

Ex. #6: Find x and y.



$b = a\sqrt{3}$

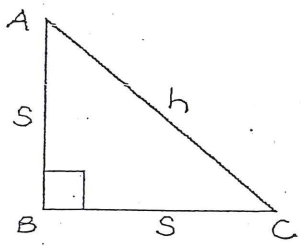
$9 = a\sqrt{3}$

$\frac{9}{\sqrt{3}} \quad \frac{\sqrt{3}}{\sqrt{3}}$

x = $\frac{9}{\sqrt{3}}$

y = $\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{18}{\sqrt{3}}$

A) Find the missing measures in Triangle ABC if...



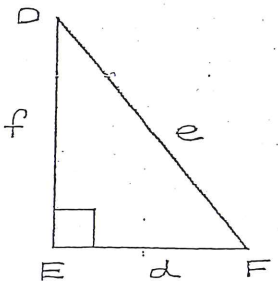
1) s = 6 h = $6\sqrt{2}$

2) h = $8\sqrt{2}$ s = 8

3) s = 4.1 h = $4.1\sqrt{2}$

4) h = $3.7\sqrt{2}$ s = 3.7

B) Find the missing measures in Triangle DEF if...



1) d = 4 f = $4\sqrt{3}$, e = 8

2) e = 10 d = 5, f = $5\sqrt{3}$

3) f = $7\sqrt{3}$ d = 7, e = 14

4) d = 9 e = 18, f = $9\sqrt{3}$